Black Holes with relativity effect (Schwarzschild Black Hole)

```
Manipulate[

 G = 1;

 M = 1;

orbitEnergy = -1/r0 + L^2/(2 r0^2) - L^2/r0^3;


 anOrbitSolution =
  Quiet@NDSolve[
    {
     r''[τ] == -(G M/r[τ]^2) + (L^2/r[τ]^3) - (3 G M L^2/r[τ]^4),

      r[0] == r0,

      r'[0] == 0,

     ϕ'[τ] == L/r[τ]^2,

     ϕ[0] == 0,

     t'[τ] == √( (r[τ]/(r[τ] - 2 G M)) +

          (r'[τ]^2 r[τ]^2/(r[τ] - 2 G M)^2) +

          (r[τ]^3 ϕ'[τ]^2/(r[τ] - 2 G M)) ),

     t[0] == 0

    },

    {r, ϕ, t},

    {τ, 0, pT}

   ];

 domain =
```

```
                (r /. anOrbitSolution[[1, 1]])["Domain"];

 {begin, end} =
                 domain[[1]];

 angleList =
                 ϕ[end] /. anOrbitSolution;

 If[symmetricOrbit == 0,
  windingNumber = angleList[[1]] / (2 π);
  ,
  windingNumber = angleList[[1]] / π;
 ];

 timeDilation = (t[end] /. anOrbitSolution)[[1]] / end;

anOrbitPlot =
ParametricPlot[
   Evaluate[
    r[τ] {Cos[ϕ[τ]], Sin[ϕ[τ]]}
     /. anOrbitSolution],
   {τ, begin, end},
   (*PlotPoints→1000,*)
   AspectRatio → 1,
   AxesOrigin → {0, 0},
   PlotRange → scale];

 anOrbitPlotReversed =
ParametricPlot[
   Evaluate[
    r[τ] {Cos[ϕ[τ]], -Sin[ϕ[τ]]}
     /. anOrbitSolution],
   {τ, begin, end},
   (*PlotPoints→1000,*)
   AspectRatio → 1,
   AxesOrigin → {0, 0},
   PlotRange → scale];



 sRadius =
  Graphics[Disk[{0, 0}, 2]];

 If[symmetricOrbit == 1,

  Show[
   {
    anOrbitPlot,
    anOrbitPlotReversed,
     sRadius,
```

```
  Graphics[
   {
    Inset[
     ToString[
      StringForm[
       "winding number  ``", windingNumber]],
     {-25, 35} scale / 38],
    Inset[
     ToString[
      StringForm[
       "time dilation       ``", timeDilation]],
     {-25, 32} scale / 38],
    Inset[
     ToString[
      StringForm[
       "orbit energy × 100      ``", 100 orbitEnergy]],
     {22, 35} scale / 38]
   }
         ]
 }, Ticks → None, ImageSize → {400, 400}],

Show[
 {
  anOrbitPlot,
  sRadius,
  Graphics[
   {
    Inset[
     ToString[
      StringForm[
       "winding number  ``", windingNumber]],
     {-25, 35} scale / 38],
    Inset[
     ToString[
      StringForm[
       "time dilation       ``", timeDilation]],
     {-25, 32} scale / 38],
    Inset[
     ToString[
      StringForm[
       "orbit energy × 100      ``", 100 orbitEnergy]],
     {22, 35} scale / 38]
   }
         ]
 }, Ticks → None, ImageSize → {400, 400}]
],
{{L , 4, "angular momentum"}, 1 / 10, 100, ImageSize → Tiny},
{{r0, 31.6, "initial radius"}, 2.5, 50, ImageSize → Tiny},
{{pT, 4850, "proper time"}, 1, 10 000, ImageSize → Tiny},
```

```
{{scale, 37.8, "view"}, 5, 100, ImageSize → Tiny},
{{symmetricOrbit, 0, "symmetric orbit"}, {0, 1}, ControlType → Checkbox},
SynchronousUpdating → False, ControlPlacement → Left, TrackedSymbols → Manipulate]
```